1

i

null space of A transpose contains [1,2-5]^T

range space is the linear combination of [3,1,1]^T and [-1,3,1]^T

ii

dimension for the null space of A transpose is 1

dimension for the range space of A is 2

iii)

the null space of A transpose is a line

the range space of A is a plane

And the two space perpendicular to each other. This can be check by noting that [1,2,-5][3,1,1]^T = 0 and [1,2,-5][-1,3,1]^T = 0

b

u is [-1,3,1]^T

v is [1,2,-5]^T

c

eigenvalues are 12 and 10 which can be obtained easily from inspection

eigenvectors are [1,1]^T and [1,-1]^T respectively

Q is [[1,1],[1,-1]] / sqrt(2)

diagonal matrix is diag(12,10)

d

i)

singulare value decomposition

U = [[1/sqrt(6), 2/sqrt(5), 1/sqrt(30)], [2/sqrt(6), -1/sqrt(5), 2/sqrt(30)], [1/sqrt(6), 0, -5/sqrt(30)]]

V = [[1,1],[1,-1]] / sqrt(2)

S = [[2sqrt(3), 0],[0,sqrt(10)], [0,0]]

ii)

{[1,1]^T, [1,-1]^T}

{[1,2,1]^T, [2,-1,0]^T, [1,2,-5]^T} the third vector is obtained using the cross product of the first two

iii)

l\_2 norm is 2sqrt(3)

e

Using the fact that singular value decomposition exist for all matrices we can write:

C = USV^T

And Considering CC^T:

(USV^T)(VS^TU^T) = US(V^TV)S^TU^T = U(SS^T)U^T

And considering C^TC:

(VS^TU^T)(USV^T) = VS^T(U^TU)SV^T = V(S^TS)V^T

We know that SS^T and S^TS is the same since they are both diagonal matrices. Then the two matrices CC^T and C^TC must have the same eigenvalues which are the diagonal elements of SS^T or S^TS.

2

a)

A^-1 = [[1,2,1.5]^T, [0,0,-0.25]^T, [0,-1,-0.75]^T]

A^-1 infinity norm = 3

A infinity norm = 7

cond(A) = 21

A^-1 1 norm = 4.5

A 1 norm = 4

cond(A) = 18

b)

i)

We use the fact that the series expansion of e^k convergent to show that the .

Since for a convergent series, its terms must tend to 0, otherwise there will be no way that the series is convergent.

ii)

Consider the metric d(x, y) = the norm of (x - y)

And it’s easy to see that the norm of (x-y) satisfies all the properties of a valid metric space.

We then show the norm of(E\_n - E\_m) is less than any epsilon that we want. And finally we can apply the Cauchy test to show that the sequence {E\_n} is Cauchy, thus the sequence is convergent. And we know that E\_n is well defined.

All you need to do is work through the algebra to show that the norm of (E\_n - E\_m) is equal to the norm of (the sum from m + 1 to n of M^i/i!). This is just using the definition of E\_n. We then use the submultiplicative properties of matrix norm and have the sum from m+1 to n of (the norm of (M))^i / i! is greater than the norm of (the sum from m + 1 to n of M^i/i!). Typing Math in here is so hard…. Basically just work through the algebra and then you can get the final result that the norm of (E\_n - E\_m) is less than any epsilon you want, since it tends to 0 (using the result we obtained from i).

And if the metric d(E\_n, E\_m) tends to 0. The sequence in the metric space is Cauchy, thus convergent.

c)

i) Minimum at (0,0) Note that using Hessian cannot obtain the fact that this is minima, since it is neither positive definitive nor negative definitive. We know this is a minimum from the fact that f(x,y) >=0 for all x and y.

ii) Saddle point at (⅓, 0) and maximum at (-3, 0)

iii) Saddle point at (0,0) and minimum at (1/12, -⅙)